INHOMOGENEOUS SHEAR FLOWS OF LINEAR POLYMERS

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Solutions of a system of equations of nonlinear viscoelastic fluid motion describing inhomogeneous shear flows of linear polymers are indicated.

The equation of motion of a nonlinear viscous fluid with some kind of nonlinearity law is often used to describe the flows of linear polymers in different devices. Such a description does not take account of some essential features of the behavior of the linear polymers and their concentrated solutions. At this time there is a need to take more specifically into account the specifics of the material in a study of the flow in elements of technological equipment.

Let us consider the governing equation which, as has been established earlier [1], describes the nonlinear effects in the flow of linear polymers and their concentrated solutions with acceptable accuracy for practice. This system of equations contains the minimum quantity of material constants and is one of the simplest nonlinear generalizations of the governing equation of a viscoelastic fluid with one relaxation time. Hence, the system under consideration can be an illustration of effects occurring in the flow of nonlinear viscoelastic materials.

Certain exact solutions of these equations, which describe flow in one direction in infinitely long cylindrical tubes of circular and annular section, the runoff layer on an inclined plane or cylindrical surfaces, Couette flow and helical flows between coaxial cylinders, are investigated in the present paper.

1. System of Equations

The equations of motion of a viscoelastic incompressible fluid [1] have the form

$$\rho\left(\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_k}\right)=\frac{\partial}{\partial x_k}\,\sigma_{ik}+F_i, \tag{1}$$

$$\frac{\partial v_k}{\partial x_k}=0, \tag{2}$$

$$\frac{d\xi_{ij}}{dt}=-\frac{1}{\tau}\left(\xi_{ij}-\frac{1}{3}\,\delta_{ij}\right)+\frac{\partial v_i}{\partial x_k}\,\xi_{kj}+\frac{\partial v_j}{\partial x_k}\,\xi_{ki}, \tag{3}$$

$$\sigma_{ij}=-p\delta_{ij}+3\,\frac{\eta_0}{\tau_0}\left(\xi_{ij}-\frac{1}{3}\,\delta_{ij}\right). \tag{4}$$

System (1)-(4) describes linear polymer flow and allows extension to filled systems on their base. Expressions (1) and (2) are the usual equations of motion of an incompressible continuous medium, (3) is the relaxation equation, and (4) is the definition of the stress tensor.

The relaxation time τ depends on the stresses applied, where the relationship

$$\tau=\tau_0 f(D),\quad D=\frac{\tau_0}{\eta_0}\,(\sigma_{ii}+3p) \tag{5}$$

is satisfied. System (1)-(5) contains two material constants: the initial coefficient of viscosity η_0 and the initial relaxation time τ_0, and one unknown monotonically decreasing universal function (5).

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It has been shown [1] by a number of examples that the mentioned system of equations describes observable singularities of linear polymer motion. The function f(D) is determined from experimental data and can be approximated for linear polymers by, say, the function

$$f(D) = \frac{1}{(1 + k\cdot D)^{\nu}} \tag{6}$$

with universal values for k and ν.

Taking (4) into account, (1)-(3) are written as follows in a cylindrical r, φ, z coordinate system:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\varphi}{r}\frac{\partial v_r}{\partial \varphi} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\varphi^2}{r}\right) + \frac{\partial p}{\partial r} = \frac{3\eta_0}{\tau_0}\left(\frac{\partial \xi_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \xi_{r\varphi}}{\partial \varphi} + \frac{\xi_{rr} - \xi_{\varphi\varphi}}{r} + \frac{\partial \xi_{rz}}{\partial z}\right) + \rho F_r, \tag{7}$$

$$\rho\left(\frac{\partial v_\varphi}{\partial t} + v_r\frac{\partial v_\varphi}{\partial r} + \frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial \varphi} + \frac{v_r v_\varphi}{r} + v_z\frac{\partial v_\varphi}{\partial z}\right) + \frac{1}{r}\frac{\partial p}{\partial \varphi} = \frac{3\eta_0}{\tau_0}\left(\frac{\partial \xi_{r\varphi}}{\partial r} + \frac{2}{r}\xi_{r\varphi} + \frac{1}{r}\frac{\partial \xi_{\varphi\varphi}}{\partial \varphi} + \frac{\partial \xi_{\varphi z}}{\partial z}\right) + \rho F_\varphi, \tag{8}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\varphi}{r}\frac{\partial v_z}{\partial \varphi} + v_z\frac{\partial v_z}{\partial z}\right) + \frac{\partial p}{\partial z} = \frac{3\eta_0}{\tau_0}\left(\frac{\partial \xi_{rz}}{\partial r} + \frac{\xi_{rz}}{r} + \frac{1}{r}\frac{\partial \xi_{\varphi z}}{\partial \varphi} + \frac{\partial \xi_{rz}}{\partial z}\right) + \rho F_z, \tag{9}$$

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r}\frac{\partial v_\varphi}{\partial \varphi} + \frac{\partial v_z}{\partial z} = 0, \tag{10}$$

$$\frac{\partial \xi_{rr}}{\partial t} + v_r\frac{\partial \xi_{rr}}{\partial r} + \frac{v_\varphi}{r}\frac{\partial \xi_{rr}}{\partial \varphi} + v_z\frac{\partial \xi_{rr}}{\partial z} - 2\left(\xi_{rr}\frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_r}{\partial \varphi}\xi_{r\varphi} + \frac{\partial v_r}{\partial z}\xi_{rz}\right) = -\frac{1}{\tau}\left(\xi_{rr} - \frac{1}{3}\right), \tag{11}$$

$$\frac{\partial \xi_{r\varphi}}{\partial t} + v_r\frac{\partial \xi_{r\varphi}}{\partial r} + \frac{v_\varphi}{r}\frac{\partial \xi_{r\varphi}}{\partial \varphi} + v_z\frac{\partial \xi_{r\varphi}}{\partial z} + \left(\frac{v_\varphi}{r} - \frac{\partial v_\varphi}{\partial r}\right)\xi_{rr} +$$
$$+ \frac{\partial v_z}{\partial z}\xi_{r\varphi} - \frac{\partial v_\varphi}{\partial z}\xi_{rz} - \frac{1}{r}\frac{\partial v_r}{\partial \varphi}\xi_{\varphi\varphi} - \frac{\partial v_r}{\partial z}\xi_{z\varphi} = -\frac{1}{\tau}\xi_{r\varphi}, \tag{12}$$

$$\frac{\partial \xi_{rz}}{\partial t} + v_r\frac{\partial \xi_{rz}}{\partial r} + \frac{v_\varphi}{r}\frac{\partial \xi_{rz}}{\partial \varphi} + v_z\frac{\partial \xi_{rz}}{\partial z} - \left(\frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z}\right)\xi_{rz} -$$
$$- \frac{\partial v_z}{\partial r}\xi_{rr} - \frac{1}{r}\frac{\partial v_z}{\partial \varphi}\xi_{r\varphi} - \frac{1}{r}\frac{\partial v_r}{\partial \varphi}\xi_{\varphi z} - \frac{\partial v_r}{\partial z}\xi_{zz} = -\frac{1}{\tau}\xi_{rz}, \tag{13}$$

$$\frac{\partial \xi_{\varphi\varphi}}{\partial t} + v_r\frac{\partial \xi_{\varphi\varphi}}{\partial r} + \frac{v_\varphi}{r}\frac{\partial \xi_{\varphi\varphi}}{\partial \varphi} + v_z\frac{\partial \xi_{\varphi\varphi}}{\partial z} + 2\left[\left(\frac{v_\varphi}{r} - \frac{\partial v_\varphi}{\partial r}\right)\xi_{r\varphi} -\right.$$
$$\left. - \frac{1}{r}\left(\frac{\partial v_\varphi}{\partial \varphi} + v_r\right)\xi_{\varphi\varphi} - \frac{\partial v_\varphi}{\partial z}\xi_{z\varphi}\right] = -\frac{1}{\tau}\left(\xi_{\varphi\varphi} - \frac{1}{3}\right), \tag{14}$$

$$\frac{\partial \xi_{zz}}{\partial t} + v_r\frac{\partial \xi_{zz}}{\partial r} + \frac{v_\varphi}{r}\frac{\partial \xi_{zz}}{\partial \varphi} + v_z\frac{\partial \xi_{zz}}{\partial z} + 2\left(-\frac{\partial v_z}{\partial r}\xi_{rz} - \frac{1}{r}\frac{\partial v_z}{\partial \varphi}\xi_{\varphi z} - \frac{\partial v_z}{\partial z}\xi_{zz}\right) = -\frac{1}{\tau}\left(\xi_{zz} - \frac{1}{3}\right), \tag{15}$$

$$\frac{\partial \xi_{\varphi z}}{\partial t} + v_r\frac{\partial \xi_{\varphi z}}{\partial r} + \frac{v_\varphi}{r}\frac{\partial \xi_{\varphi z}}{\partial \varphi} + v_z\frac{\partial \xi_{\varphi z}}{\partial z} + \left(\frac{v_\varphi}{r} - \frac{\partial v_\varphi}{\partial r}\right)\xi_{rz} +$$
$$+ \frac{\partial v_r}{\partial r}\xi_{\varphi z} - \frac{\partial v_\varphi}{\partial z}\xi_{zz} - \frac{\partial v_z}{\partial r}\xi_{\varphi r} - \frac{1}{r}\frac{\partial v_z}{\partial \varphi}\xi_{\varphi\varphi} = -\frac{1}{\tau}\xi_{\varphi z}, \tag{16}$$

$$D = 3(\xi_{rr} + \xi_{\varphi\varphi} + \xi_{zz} - 1). \tag{17}$$

Stationary solutions of the system of equations (5), (7)-(17) are henceforth examined. The vector of the mass forces F_i is zero everywhere except where specially stipulated.

2. Rectilinear Flows

We seek an axisymmetric solution, independent of the axial coordinate z, for the system of equations (5), (7)-(17) in which

$$v_z = u(r),\quad \xi_{rz} = \xi_1(r),\quad D = D(r),\quad v_r = v_\varphi = \xi_{r\varphi} = \xi_{z\varphi} = 0,$$
$$\frac{\partial p}{\partial z} = -A = \text{const}. \tag{18}$$

The continuity equation (10) is here satisfied identically, $\xi_{rr} = \xi_{\varphi\varphi} = 1/3$, $\xi_{zz} = {}^1/_3\cdot(1 + D)$, and the equation of motion (9) yields

$$\xi_1(r) = \frac{\tau_0}{3\eta_0}\left(-\frac{A}{2}r + \frac{B}{r}\right). \tag{19}$$

The relationships

$$-\frac{1}{3}\frac{du}{dr} = -\frac{1}{\tau(D)}\xi_1(r),\quad -2\frac{du}{dr}\xi_1(r) = -\frac{1}{\tau(D)}\left(\xi_{zz} - \frac{1}{3}\right), \tag{20}$$

from which we obtain

$$u(r) = \frac{3}{\tau_0}\int \frac{\xi_1(r)}{f(D(r))}dr + C_1,\quad D = 18\xi_1^2, \tag{21}$$

are a corollary of the relaxation equations. The pressure is found by integrating (7):

$$p = p_0 - Az. \tag{22}$$

The first and second differences of the normal stresses are, respectively, for the class of flows under consideration

$$\sigma_1 \equiv \sigma_{zz} - \sigma_{rr} = 2\frac{\tau_0}{\eta_0}\left(\frac{1}{4}A^2r^2 - AB + \frac{B^2}{r^2}\right),\quad \sigma_2 \equiv \sigma_{rr} - \sigma_{\varphi\varphi} = 0. \tag{23}$$

Therefore, the desired general solution is given by (18), (19), (21)-(23). The constants in this solution are found from the boundary conditions. Let us examine some particular cases. Let the flow occur in a circular infinite tube of radius R under the effect of a constant pressure gradient dp/dz = –A. Then, from the condition of boundedness of the shear stress on the axis of symmetry of the tube B = 0. Taking the condition of adhesion on the tube surface, we obtain the following expressions for the distributions of the axial velocity, the volume mass flow rate, and the first difference of the normal stresses

$$u(r) = \frac{\eta_0}{2A\tau_0^2}\int_{D(r)}^{D(R)}\frac{ds}{f(s)},\quad Q = \frac{\pi\eta_0^3}{A^3\tau_0^4}\int_0^{D(R)}ds\int_s^{D(R)}\frac{dt}{f(t)},$$
$$\sigma_1 = \frac{A^2\tau_0}{2\eta_0}r^2,\quad D(r) = \frac{A^2\tau_0^2}{2\eta_0^2}r^2. \tag{24}$$

If the approximation of f(D) in the form (6) is used for $\nu = 1$, then (24) becomes

$$u = \frac{A}{4\eta_0}(R^2 - r^2)\left[1 + \frac{kA^2\tau_0^2}{4\eta_0^2}(R^2 + r^2)\right], \tag{25}$$

$$Q = \frac{\pi A R^4}{8\eta_0}\left(1 + \frac{kA^2\tau_0^2}{3\eta_0^2}R^2\right). \tag{26}$$

The first terms in (25) and (26) are known for a viscous fluid, the second yield the correction for non-Newtonian viscoelastic behavior. In conformity with (26), a deviation from the viscous fluid flow law [2, 3] is observed for linear polymers with a filler. Relationship (26) can be used to determine the material constants η_0, τ_0 by means of the experimental dependence of the flow rate on the applied pressure gradient.

Among the solutions of the form (18), (19), (21)-(23) there is a solution describing the forced rectilinear flow between two coaxial cylinders when one of the cylinders (the outer, say) is fixed while the inner moves along its axis at a constant velocity. A flow of this kind occurs during the deposition of different coatings on a wire.

The general solution (18), (19), (21)-(23) also describes the flow occurring in an annular channel formed by two fixed concentric cylinders subjected to a given pressure drop. If it is assumed, as before, that the motion is steady, rectilinear, and axisymmetric, then the constant A in (19) is known, and B is found from the adhesion condition. The specific form of the solution is contained as a particular case in the relationships presented below for the helical flow of a nonlinear viscoelastic fluid between coaxial cylinders.

The exact solution of (7)-(16) examined in this section can be interpreted, for an appropriate selection of constants, as the flow in a thin film. Indeed, let the laminar flow of a viscoelastic fluid be in a thin film of constant thickness h along the surface (outer, say) of a vertical circular cylinder in a gravity force field. If the acceleration of gravity acts in the positive direction of the z axis, then $A = \rho g$. From the condition that the tangential stress of the film free surface is zero, we find $B = (A/2)(R+h)^2$. The solution has the form

$$u(r) = \frac{\eta_0}{2A\tau_0^2} \int_{D(R)}^{D(r)} \frac{\frac{s}{2\varkappa} + (R+h)^2 - \sqrt{\left(\frac{s}{2\varkappa}\right)^2 + \frac{s}{\varkappa}(R+h)^2}}{\frac{s}{2\varkappa} + 2(R+h)^2 - \sqrt{\left(\frac{s}{2\varkappa}\right)^2 + \frac{s}{\varkappa}(R+h)^2}} \frac{ds}{f(s)},$$

$$Q = \frac{\pi\eta_0^3}{2A^3\tau_0^4} \int_0^{D(R)} \left(1 - \frac{\frac{s}{2\varkappa} + (R+h)^2}{\sqrt{\left(\frac{s}{2\varkappa}\right)^2 + \frac{s}{\varkappa}(R+h)^2}}\right) ds \times$$

$$\times \int_{D(R)}^{s} \frac{\frac{t}{2\varkappa} + (R+h)^2 - \sqrt{\left(\frac{t}{2\varkappa}\right)^2 + \frac{t}{\varkappa}(R+h)^2}}{\frac{t}{2\varkappa} + 2(R+h)^2 - \sqrt{\left(\frac{t}{2\varkappa}\right)^2 + \frac{t}{\varkappa}(R+h)^2}} \frac{dt}{f(t)}, \tag{27}$$

$$\sigma_{rz} = \frac{A}{2r}((R+h)^2 - r^2), \quad \sigma_1 = \frac{\eta_0}{\tau_0} D, \quad D = \varkappa\left(\frac{(R+h)^2 - r^2}{r}\right)^2,$$

$$\varkappa = \frac{1}{2}\left(\frac{A\tau_0}{\eta_0}\right)^2.$$

By passing to the limit as $R \to \infty$ and for fixed h in (27), we obtain formulas describing the runoff of a film of thickness h on an inclined plane making the angle α with the horizon:

$$u(y) = -\frac{\eta_0}{4A\tau_0^2} \int_{D(0)}^{D(y)} \frac{ds}{f(s)}, \quad Q = \frac{\eta_0^2}{8\sqrt{2}A^2\tau_0^3} \int_{D(0)}^{D(h)} \frac{ds}{\sqrt{s}} \int_{D(0)}^{s} \frac{dt}{f(t)},$$

$$\sigma_{xy} = \frac{A}{2}(h-y), \quad \sigma_1 = \frac{\eta_0}{\tau_0} D, \quad D = 4\varkappa(h-y)^2,$$

$$p = p_0 + \rho g(h-y)\cos\alpha.$$

Here $A = \rho g \sin\alpha$; and y is the distance to the inclined plane. For the universal function defined by (6) and $\nu = 1$, we obtain

$$u(y) = \frac{A}{2\eta_0} y(2h-y)\left(1 + k\left(\frac{A\tau_0}{\eta_0}\right)^2((h-y)^2 + h^2)\right),$$

$$Q = \frac{h^3 A}{3\eta_0}\left(1 + \frac{6}{5}k\left(\frac{A\tau_0}{\eta_0}\right)^2 h^2\right).$$

3. Curvilinear Flows

Let us consider the stationary solution of the initial system of equations (7)-(16) that describes steady circular motion over concentric circles. It is sought in the form

$$v_\varphi = \mathrm{v}(r), \quad \xi_{r\varphi} = \xi_2(r), \quad D = D(r), \quad v_r = v_z = \xi_{rz} = \xi_{z\varphi} = 0. \tag{28}$$

The continuity equation is satisfied identically in this solution. We obtain from the relaxation equations (11) and (15) and the equation of motion (8)

$$\xi_2 = \frac{C}{r^2}, \quad \xi_{rr} = \xi_{zz} = \frac{1}{3}, \quad \xi_{\varphi\varphi} = \frac{1}{3}(1 + D). \tag{29}$$

The relaxation equations (12) and (14) yield

$$\frac{1}{3}\left(\frac{v}{r} - \frac{dv}{dr}\right) = -\frac{\xi_2}{\tau}, \quad 2\left(\frac{v}{r} - \frac{dv}{dr}\right)\xi_2 = -\frac{D}{3\tau},$$

from which we obtain

$$v(r) = \frac{3}{\tau_0} r \int \frac{\xi_2(r)\,dr}{rf(D(r))} + C_2 r, \quad D = 18\xi_2^2. \tag{30}$$

We obtain the pressure by integrating (7):

$$p = p_0 + \rho \int \frac{v^2(r)}{r}\,dr - \frac{\eta_0}{\tau_0}\int \frac{D(r)}{r}\,dr. \tag{31}$$

The first and second differences of the normal stresses equal, respectively:

$$\sigma_1 \equiv \sigma_{\varphi\varphi} - \sigma_{rr} = \frac{\eta_0}{\tau_0} D, \quad \sigma_2 \equiv \sigma_{rr} - \sigma_{zz} = 0. \tag{32}$$

The constants in solution (28)-(32) are found from the boundary conditions.

Let us turn to Couette flow that is realized in an annular slot between coaxial cylinders, one of which (the outer) is at rest while the other rotates at the angular velocity Ω = const. Let the torque (which can easily be measured) per unit length be M = const. Then

$$v(r) = -\frac{1}{4\sqrt{2}\,\tau_0} r \int_{D(r)}^{D(R_2)} \frac{ds}{\sqrt{s}\,f(s)}, \quad C = \frac{M}{6\pi}\frac{\tau_0}{\eta_0}. \tag{33}$$

The angular velocity is related to the torque by the relationship

$$\Omega = -\frac{1}{4\sqrt{2}\,\tau_0} \int_{D(R_1)}^{D(R_2)} \frac{ds}{\sqrt{s}\,f(s)}, \quad D(r) = \frac{3M\tau_0}{\pi\eta_0}\frac{1}{r^2}. \tag{34}$$

which can be used to find the rheological characteristics η_0, τ_0 of a nonlinear viscoelastic fluid from the experimental dependence of the cylinder angular velocity on the torque measured on a rotation viscosimeter.

If f(D) is defined by (6) and ν = 1, then (33) and (34) become

$$\begin{gathered} v(r) = \frac{M}{4\pi\eta_0} r \left[\left(\frac{1}{r^2} - \frac{1}{R_2^2}\right) + \frac{k}{6}\left(\frac{M\tau_0}{\eta_0}\right)^2\left(\frac{1}{r^6} - \frac{1}{R_2^6}\right)\right], \\ \Omega = \frac{M}{4\pi\eta_0}\left[\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right) + \frac{k}{6}\left(\frac{M\tau_0}{\eta_0}\right)^2\left(\frac{1}{R_1^6} - \frac{1}{R_2^6}\right)\right]. \end{gathered} \tag{35}$$

Exactly as in (25) and (26), the second terms here yield the correction for non-Newtonian viscoelastic behavior.

Now, let us mention the more general solution that includes all the flows considered above as particular cases. It has the form

$$\begin{gathered} v_\varphi = v(r), \quad v_z = u(r), \quad \xi_{rz} = \xi_1(r), \quad \xi_{r\varphi} = \xi_2(r), \\ \xi_{\varphi z} = \xi_3(r), \quad \xi_{rr} = \frac{1}{3}, \quad p = p_0 + \rho\int \frac{v^2(r)}{r}\,dr - 18\frac{\eta_0}{\tau_0}\int \frac{\xi_2^2(r)}{r}\,dr - Az, \end{gathered} \tag{36}$$

where

$$\begin{gathered} u(r) = \frac{3}{\tau_0}\int \frac{\xi_1(r)}{f(D(r))}\,dr + C_1; \quad v(r) = \frac{3}{\tau_0} r \left(\int \frac{\xi_2(r)}{rf(D(r))}\,dr + C_2\right); \\ D(r) = 18(\xi_1^2(r) + \xi_2^2(r)); \quad \xi_3 = -6\xi_1\xi_2; \\ \sigma_1 \equiv \sigma_{zz} - \sigma_{rr} = 18\frac{\eta_0}{\tau_0}\xi_1^2; \quad \sigma_2 \equiv \sigma_{\varphi\varphi} - \sigma_{rr} = 18\frac{\eta_0}{\tau_0}\xi_2^2; \end{gathered} \tag{37}$$

$$\xi_1 = \frac{\tau_0}{3\eta_0}\left(-\frac{A}{2}r + \frac{B}{r}\right); \quad \xi_2 = \frac{C}{r^2}.$$

Analogously to the preceding, the constants A, B, C are determined from the boundary conditions on the surfaces r = const. A solution of form (37) describes helical flow.

Formulas (36) and (37) can be used as tests for the execution of numerical computations of nonlinear viscoelastic fluid flows, as well as for the approximate description of spiral flows of linear polymers with a filler. The solutions obtained can be used also to describe polymer flows in the production of profiled items, heat carrier flow in heat exchange systems, viscosimeter flows. And, finally, they are useful as the simplest describers of nonlinear effects and can be used as etalons for the development of numerical methods to solve problems on nonlinear viscoelastic fluid motion.

NOTATION

σ_{ij}, stress tensor, p, pressure; F_i, mass force vector; δ_{ij}, Kronecker delta; η, coefficient of shear viscosity; τ, relaxation time; ξ_{ij}, inner parameter; $\nu_{ij} = \partial v_i/\partial x_j$, velocity gradient tensor; η_0, initial value of the shear viscosity coefficient; τ_0, initial value of the relaxation time; D, dimensionless first invariant of the additional stress tensor; A, B, C, constants of integration; f(D), universal function characterizing the material; r, φ, z, cylindrical coordinates; $u = v_z$, axial component of the velocity vector; v = v , circumferential component of the velocity vector; σ_1, σ_2, first and second differences of the normal stress; Q, volume mass flow rate; R, radius of a circular tube; R_1, R_2, radii of the inner and outer cylinders, respectively; M, moment per unit length.

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